Inequalities

Questions

Q1.

Use algebra to find the set of values of *x* for which

$$x \geqslant \frac{2x+15}{2x+3}$$

(Total for question = 6 marks)

Q2.

Use algebra to find the set of values of *x* for which

$$\frac{x-2}{2(x+2)} \leqslant \frac{12}{x(x+2)}$$

(9)

(Total for question = 9 marks)

Q3.

A student was set the following problem.

Use algebra to find the set of values of x for which

$$\frac{x}{x-24} > \frac{1}{x+11}$$

The student's attempt at a solution is written below.

$$x(x-24)(x+11)^2 > (x+11)(x-24)^2$$

$$x(x-24)(x+11)^2 - (x+11)(x-24)^2 > 0$$

$$(x-24)(x+11)\left[x(x+11) - x - 24\right] > 0 \qquad \text{Line 3}$$

$$(x-24)(x+11)\left[x^2 + 10x - 24\right] > 0$$

$$(x-24)(x+11)(x+12)(x-2) > 0$$

$$x = 24,\ x = -11,\ x = -12,\ x = 2$$

$$\{x \in \mathbb{R} : -12 < x < -11\} \cup \{x \in \mathbb{R} : 2 < x < 24\} \qquad \text{Line 7}$$

There are errors in the student's solution.

(a) Identify the error made

(i) in line 3
(ii) in line 7

(2)

(b) Find a correct solution to this problem.

(4)

(Total for question = 6 marks)

Q4.

Use algebra to determine the values of *x* for which

$$\frac{x+1}{2x^2+5x-3} > \frac{x}{4x^2-1}$$

(Total for question = 5 marks)

Q5.

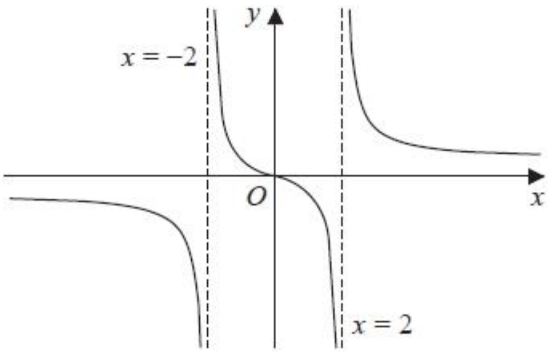


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{|x| - 2}$$

Use algebra to determine the values of *x* for which

$$2x - 5 > \frac{x}{|x| - 2}$$

(8)

(Total for question = 8 marks)

Q6.

Use algebra to determine the values of *x* for which

$$x(x-1) > \frac{x-1}{x}$$

giving your answer in set notation.

(Total for question = 6 marks)

Q7.

Use algebra to find the set of values of *x* for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

(Total for question = 6 marks)

Q8.

Use algebra to find the values of *x* for which

$$\frac{x}{x^2-2x-3} \leqslant \frac{1}{x+3}$$

(7)

(Total for question = 7 marks)

Mark Scheme - Inequalities

Q1.

Question	Scheme	Marks	AOs
	$x=\frac{2x+15}{2x+3}\Rightarrow 2x^2+3x=2x+15\Rightarrow 2x^2+x-15=0\Rightarrow x=...$ Alternative 1: $(2x+3)^2x\geqslant(2x+3)(2x+15)\Rightarrow(2x+3)(2x^2+3x-2x-15)\geqslant0$ $(2x+3)(x+3)(2x-5)\geqslant0$ Alternative 2; $x-\frac{2x+15}{2x+3}\geqslant0\Rightarrow\frac{x(2x+3)-2x-15}{2x+3}\geqslant0\Rightarrow\frac{(x+3)(2x-5)}{2x+3}\geqslant0$	M1	1.1b
	$\Rightarrow(x+3)(2x-5)=0\Rightarrow$ CVs are $-3,\ \frac{5}{2}$	A1	1.1b
	Also $2x+3=0\Rightarrow x=-\frac{3}{2}$ a CV	B1	2.3
	Hence from graph (oe) the solution set is $\left\{x\in\mathbb{R}:-3\leqslant x<-\frac{3}{2},x\geqslant\frac{5}{2}\right\}\left\{x:-3\leqslant x<-\frac{3}{2},x\geqslant\frac{5}{2}\right\}$	M1 A1 A1	1.1b 2.2a 2.5
		(6)	
			(6 marks)

Notes:

M1: For a complete method to find the critical values other than $-\frac{3}{2}$.

Alternative 1: Multiplies by $(2x+3)^2$, collects terms onto one side and factorises into three brackets.

Alternative 2: Collects terms onto one side and combines into single fraction using a common denominator and factorises the numerator

A1: Correct critical values -3 and $\frac{5}{2}$

B1: For the critical value $-\frac{3}{2}$

M1: Selects the correct regions for their three CV's. Should include the right hand side open ended and another bounded region. CV's of a < b < c then must be of the form $a\leqslant x\leqslant b, x\geqslant c$ or $a<x<b, x>c$ the direction of the inequalities must be correct with or without strict inequalities.

A1: At least one correct interval identified. Alternatively allow for both intervals with correct end points but incorrect strict or inclusive inequalities

A1: Fully correct solution as a set – accept alternative set notations e.g. $\left[-3,-\frac{3}{2}\right)\cup\left[\frac{5}{2},\infty\right)$, but not just inequalities. Minimum use of set notation $-3\leqslant x<-\frac{3}{2}\cup x\geqslant\frac{5}{2}$

Note: Correct answer with no working scores M0 A0 but can score B1 M1 A1 A1

No working shown to factorise a cubic equation e.g.
$4x^3+8x^2-27x-45=(x+3)(2x+3)(2x-5)$ is M0 A0 but can still score B1 M1 A1 A1

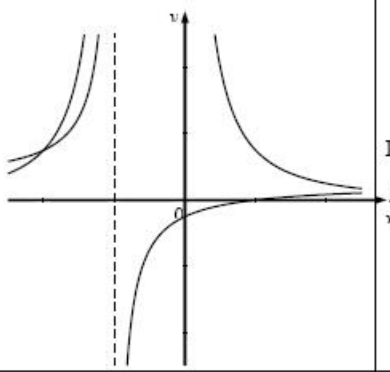
A0 for $-3\leqslant x<-\frac{3}{2}\cap x\geqslant\frac{5}{2}$ or $-3\leqslant x<-\frac{3}{2}$ and $x\geqslant\frac{5}{2}$

Special case: If they have a repeated root final 3 marks M1 A1 A0 is possible e.g.

Q2.

Question Number	Scheme	Notes	Marks
	$\dfrac{x-2}{2(x+2)} \le \dfrac{12}{x(x+2)}$		
NB	Question states "Use algebra..." so purely graphical solutions score max 1/9 (the B1). A sketch and some algebra to find CVs or intersection points can score according to the method used.		
	Can use $\le, <$ or $=$ for the first 6 marks **in all methods**		
	$\dfrac{x-2}{2(x+2)} - \dfrac{12}{x(x+2)} (\le 0)$	Collects expressions to one side.	M1
	$\dfrac{x^2-2x-24}{2x(x+2)} (\le 0)$	M1: Attempt common denominator A1: Correct single fraction	M1A1
	$x = 0, -2$	Correct critical values	B1
	$x^2-2x-24 \Rightarrow (x+4)(x-6)(=0) \Rightarrow x = ...$	Attempt to solve their quadratic as far as $x = ...$	M1
	$x = -4, 6$	Correct critical values. May be seen on a sketch.	A1
	$-4 \le x < -2, \ 0 < x \le 6$ with $\le$ or $<$ throughout	M1: Attempt two inequalities using their 4 critical values in ascending order. (dependent on at least one previous M mark) A1: All 4 CVs in the inequalities correct	dM1A1
	$-4 \le x < -2, \ 0 < x \le 6$ $[-4, -2) \cup (0, 6]$	A1:Inequality signs correct Set notation may be used. $\cup$ or "or" but not "and"	A1cao (9)
			Total 9

	Alternative 1: Multiplies both sides by $x^2(x+2)^2$		
	$x^2(x-2)(x+2) \le 24x(x+2)$ $x^3(x+2) - 2x^2(x+2) \le 24x(x+2)$	Both sides $\times x^2(x+2)^2$ May multiply by more terms but must be a positive multiplier containing $x^2(x+2)^2$	M1
	$x^3(x+2) - 2x^2(x+2) - 24x(x+2) (\le 0)$	M1: Collects expressions to one side A1: Correct inequality	M1A1
	$x = 0, -2$	Correct critical values	B1
	$x^4 - 28x^2 - 48x (= 0)$ $x(x+2)(x-6)(x+4)(=0) \Rightarrow x = ...$	Attempt to solve their quartic as far as x = ... to obtain the **other** critical values Can cancel x and solve a cubic or x and $(x+2)$ and solve a quadratic.	M1
	$x = -4, 6$	Correct critical values	A1
	$-4 \le x < -2, \ 0 < x \le 6$ with $\le$ or $<$ throughout	M1: Attempt two inequalities using their 4 critical values in ascending order. (dependent on at least one previous M mark) A1: All 4 CVs in the inequalities correct	dM1A1
	$-4 \le x < -2, \ 0 < x \le 6$ $[-4, -2) \cup (0, 6]$	A1:Inequality signs correct Set notation may be used. $\cup$ or "or" but not "and"	A1cao (9)
			Total 9

	Alternative 2: using a sketch graph (probably from calculator)		
		Draw graphs of $y = \frac{x-2}{2(x+2)}$ and $y = \frac{12}{x(x+2)}$	
	CVs $x = 0, -2$	(Vertical asymptotes of graphs.)	B1
	$\frac{x-2}{2(x+2)} = \frac{12}{x(x+2)}$	Eliminate y	M1
	$x(x-2) = 24$	M1: Obtains a quadratic equation A1: Correct equation	M1A1
	$x^2 - 2x - 24 \Rightarrow (x+4)(x-6) = 0 \Rightarrow x = ..$	Attempt to solve their quadratic as far as $x = ...$	M1
	CVs $x = -4, 6$	Correct critical values	A1
	$-4 \le x < -2,\ 0 < x \le 6$ with $\le$ or $<$ throughout	M1: Attempt two inequalities using their 4 critical values in ascending order. (dependent on at least one previous M mark)	dM1
	$-4 \le x < -2,\ 0 < x \le 6$	A1: All 4 CVs in the inequalities correct	A1
		A1: All inequality signs correct	A1cao (9)
NB	As above, but with no sketch graph shown: CVs $x = 0, -2$ **must** be stated somewhere.		B1
	Otherwise no marks available.		

Q3.

Question	Scheme	Marks	AOs
(a)(i)	Line 3: Allow any of either • bracketing error • -24 should be 24 in the square brackets • $x(x+11)-x-24$ should be $x(x+11)-(x-24)$ • $x(x+11)-x-24$ should be $x(x+11)-x+24$	B1	2.3
(a)(ii)	Line 7: Allow any of either • should be $\{x \in \mathbb{R} : x < -12 \text{ or } -11 < x < 2 \text{ or } x > 24\}$ • they have found the regions where the inequality is < 0 • they have reversed the inequality	B1	2.3
		(2)	

(b) Way 1	$(x-24)(x+11)[x(x+11)-(x-24)]>0$ $(x-24)(x+11)[x^2+10x+24]>0$	M1	1.1b
	$(x-24)(x+11)(x+6)(x+4)>0$ Critical values $x=-11,\ -6,\ -4,\ 24$	A1	1.1b
	$\{x\in\mathbb{R}:x<-11\}\cup\{x\in\mathbb{R}:-6<x<-4\}\cup\{x\in\mathbb{R}:x>24\}$	M1 A1	2.2a 2.5
		(4)	
(b) Way 2	$\dfrac{x}{x-24}>\dfrac{1}{x+11}\Rightarrow\dfrac{x}{x-24}-\dfrac{1}{x+11}>0\Rightarrow\dfrac{x(x+11)-(x-24)}{(x-24)(x+11)}>0$	M1	1.1b
	$\Rightarrow\dfrac{x^2+10x+24}{(x-24)(x+11)}>0\ \Rightarrow\ \dfrac{(x+6)(x+4)}{(x-24)(x+11)}>0$ Critical values $x=-11,\ -6,\ -4,\ 24$	A1	1.1b
	$\{x\in\mathbb{R}:x<-11\}\cup\{x\in\mathbb{R}:-6<x<-4\}\cup\{x\in\mathbb{R}:x>24\}$	M1 A1	2.2a 2.5
		(4)	
(b) Way 3	Considering $x<-11$ $\dfrac{x}{x-24}>\dfrac{1}{x+11}\Rightarrow x^2+11x>x-24\Rightarrow x^2+10x+24>0$ gives $x<-6$ or $x>-4$. Hence $x<-11$ Considering $-11<x<24$ $\dfrac{x}{x-24}>\dfrac{1}{x+11}\Rightarrow x^2+11x<x-24\Rightarrow x^2+10x+24<0$	M1	1.1b
	gives $-6<x<-4$. Hence $-6<x<-4$ Considering $x>24$ $\dfrac{x}{x-24}>\dfrac{1}{x+11}\Rightarrow x^2+11x>x-24\Rightarrow x^2+10x+24>0$ gives $x<-6$ or $x>-4$. Hence $x>24$	A1	1.1b
	Overall, $\{x\in\mathbb{R}:x<-11\}\cup\{x\in\mathbb{R}:-6<x<-4\}\cup\{x\in\mathbb{R}:x>24\}$	M1 A1	2.2a 2.5
		(4)	
			(6 marks)

Notes for Question	
(a)(i)	
B1:	See scheme
Note:	Give B0 for contradictory reasons
(a)(ii)	**Way 1**
B1:	See scheme
Note:	Give B0 for contradictory reasons
Note:	Allow "Should be $x<-12$, $-11<x<2$, $x>24$ "
Note:	Do not allow • "Should be $x<-12 \cap -11<x<2 \cap x>24$ " • They have found where $x<0$ and not where $x>0$ • "There should be 3 inequalities and not 2 inequalities" • "The sign is the wrong way around"
(b)	**Way 1**
M1:	Uses brackets {to correct the error made on line 3}, forms a 3TQ and uses a correct method of solving a 3TQ to give $x=...$
A1:	All four correct critical values for x
M1:	Deduces that the 2 "outsides" and the "middle interval" are required
A1:	Exactly 3 correct intervals. Their answer must be given in set notation. Accept equivalent set notation. E.g. Allow • $\{x\in\mathbb{R}: x<-11 \text{ or } -6<x<-4 \text{ or } x>24\}$ • $\{x<-11 \text{ or } -6<x<-4 \text{ or } x>24\}$ • $\{x<-11 \cup -6<x<-4 \cup x>24\}$ • $\mathbb{R}-([-11,-6]\cup[-4,24])$
Note:	Give final A0 for $\{x\in\mathbb{R}: x<-11\}\cap\{x\in\mathbb{R}: -6<x<-4\}\cap\{x\in\mathbb{R}: x>24\}$
Note:	Allow A1 for $\{x\in\mathbb{R}: x<-11, -6<x<-4, x>24\}$

(b)	**Way 2**
M1:	Gathers terms on one side and puts over a common denominator. Simplifies the numerator to $x(x+11)-(x-24)$ {and thereby corrects the error made in line 3}, forms a 3TQ and uses a correct method of solving a 3TQ to give $x=...$
A1:	See Way 1
M1:	See Way 1
A1:	See Way 1
(b)	**Way 3**
M1:	Considers each of the intervals $x<-11$, $-11<x<24$, $x>24$ separately and evaluates which parts (if any) of these regions satisfy the original inequality
A1:	Obtains a correct inequality statement for each of the intervals $x<-11$, $-11<x<24$, $x>24$
M1:	See Way 1
A1:	See Way 1

Q4.

Question	Scheme	Marks	AOs
	$\dfrac{x+1}{2x^2+5x-3} > \dfrac{x}{4x^2-1}$		
	$\dfrac{2x^2+3x+1-x^2-3x}{(2x-1)(2x+1)(x+3)} > 0$ or $(x+1)(2x-1)(2x+1)^2(x+3) - x(2x-1)(2x+1)(x+3)^2 > 0$	M1	2.1
	$\dfrac{x^2+1}{(2x-1)(2x+1)(x+3)} > 0$ or $(x+3)(2x-1)(2x+1)(x^2+1) > 0$	dM1	1.1b
	All three critical values $-3, -\frac{1}{2}, \frac{1}{2}$	A1	1.1b
	$\left\{x \in \mathbb{R} : -3 < x < -\frac{1}{2}\right\} \cup \left\{x \in \mathbb{R} : x > \frac{1}{2}\right\}$	dM1 A1	2.2a 2.5
		(5)	
		(5 marks)	

Notes

M1: Gathers terms on one side and puts over a common denominator, or multiplies by $(2x+1)^2(2x-1)^2(x+3)^2$ and gathers terms on one side

dM1: Expands and simplifies numerator or factorises into 4 factors. Depends on the previous method mark.

A1: Correct critical values and no "extras" but ignore any attempts to solve $x^2 + 1 = 0$ (correct or otherwise)

dM1: Deduces that 1 "inside" inequality and 1 "outside" inequality is required with critical values in ascending order. Depends on the previous method mark.

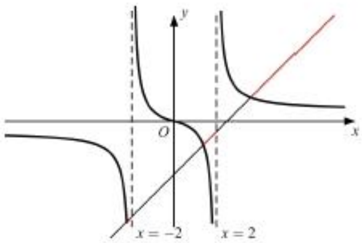
A1: Exactly 2 correct intervals, accepting equivalent notation

Special Case: Allow M1M0A0M0A0

$$\frac{x+1}{2x^2+5x-3} > \frac{x}{4x^2-1} \Rightarrow \frac{x+1}{(2x-1)(x+3)} > \frac{x}{(2x-1)(2x+1)} \Rightarrow \frac{x+1}{(x+3)} > \frac{x}{(2x+1)}$$

$\Rightarrow (x+1)(x+3)(2x+1)^2 > x(x+3)^2(2x+1)$ etc.

Q5.

Question	Scheme	Marks	AOs
	For $x<0$ need $2x-5>\frac{x}{-x-2}$ and for $x\geqslant 0$ need $2x-5>\frac{x}{x-2}$ and goes on to find the critical values for each.	M1	3.1a
	For $x\geqslant 0$: $2x-5=\frac{x}{x-2}\Rightarrow 2x^2-10x+10=0\Rightarrow x=...$	M1	1.1b
	$x=\frac{5\pm\sqrt{5}}{2}$ (oe) awrt 3.62 and awrt 1.38	A1	1.1b
	For $x<0$: $2x-5=\frac{x}{-x-2}\Rightarrow -2x^2+10=0\Rightarrow x=...$	M1	1.1b
	$x=-\sqrt{5}$ only ($\sqrt{5}$ must be rejected at some stage)	A1	2.3
	 Uses graph or other means to identify correct regions. Asymptotes must have been considered, but may miss the region near $x=-2$ So e.g. "$-\sqrt{5}<x<-2$" or "$\frac{5-\sqrt{5}}{2}<x<2$" or "$x>\frac{5+\sqrt{5}}{2}$"	M1	3.1a
	Inequality holds when $-\sqrt{5}<x<-2$ or $\frac{5-\sqrt{5}}{2}<x<2$ or $x>\frac{5+\sqrt{5}}{2}$ Accept equivalent notation, e.g $(-\sqrt{5},-2)\cup\left(\frac{5-\sqrt{5}}{2},2\right)\cup\left(\frac{5+\sqrt{5}}{2},\infty\right)$	A1ft A1	2.2a 2.5
		(8)	
			(8 marks)

Notes:

M1: Considers the two cases of $x<0$ and $x\geqslant 0$ to find critical values. Don't be concerned which side the $x=0$ case is considered part of. Allow if "=" used when considering C.V.s. This mark is for the overall strategy, so both cases must be considered, or equivalent complete longer methods.

M1: Correct method for intersection of line and curve for x positive.

A1: Line and curve intersect at $x=\frac{5\pm\sqrt{5}}{2}$

M1: Correct method for intersection of line and curve for x negative.

A1: Line and curve intersect at $x=-\sqrt{5}$ Must have rejected the positive value for this mark (though may be done later)

M1: Uses the graph (or other method) to identify at least one correct region, which must include consideration of the vertical asymptotes. Implied by two correct intervals being given for their critical values. Allow if $y=2x-5$ is added to the sketch and at least two (not necessarily correct) intervals produced as long as the points $x=\pm 2$ are excluded.

A1ft: At least one correct interval identified following through their solutions (as long as it is sensible).

A1: Fully correct solution, all three intervals given – accept alternative notations, may be just listed (no need for unions shown).

Multiplying both sides by $(x-2)^2$ or $(|x|-2)^2$ can score a maximum of M0 M1 A1 M0 A1 M1 A1ft A0

M0 M1: for multiplying through by $(x-2)^2$

$(2x-5)(x-2)^2 > x(x-2)$

$(x-2)\left[(2x-5)(x-2)-x\right] > 0$ leading to a value for x

$(x-2)(x^2-5x+5) > 0$

A1: Line and curve intersect at $x=\dfrac{5\pm\sqrt{5}}{2}$

M0A0: Not finding the point of intersection for negative x

M1 A1ft: for either "$x>\dfrac{5+\sqrt{5}}{2}$" or "$\dfrac{5-\sqrt{5}}{2}<x<2$"

A0:

If they multiply through by $(-x-2)^2$ the other marks can be scored

Q6.

Question	Scheme	Marks	AOs
	$x(x-1) > \frac{x-1}{x}$		
	$\frac{x^2(x-1)-x-1}{x}>0$ or $x^3(x-1)-x(x-1)>0$	M1	2.1
	$\frac{(x-1)^2(x+1)}{x}>0$ or $x(x-1)^2(x+1)>0$	M1	1.1b
	Critical values 0 and 1	A1	1.1b
	All three critical values $-1,\ 0,\ 1$	A1	1.1b
	$\{x\in\mathbb{R}:x<-1\}\cup\{x\in\mathbb{R}:0<x<1\}\cup\{x\in\mathbb{R}:x>1\}$	M1 A1	2.2a 2.5
		(6)	
			(6 marks)

Notes

M1: Gathers terms on one side and puts over a common denominator, or multiplies by x^2 and gathers terms on one side
M1: Factorises numerator into 3 factors or factorises into 4 factors
A1: Identifies the critical values 0 and 1
A1: All 3 correct critical values
M1: Deduces that 1 "inside" inequality and 2 "outside" inequalities are required with critical values in ascending order as shown
A1: Exactly 3 correct intervals using correct notation
Allow e.g. $\{x:x<-1\}\cup\{x:0<x<1\}\cup\{x:x>1\}$

Q7.

Question	Scheme	Marks	AOs
	$\frac{1}{x} < \frac{x}{x+2}$		
	$\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$	M1	2.1
	$\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$	M1	1.1b
	At least two correct critical values from $-2, -1, 0, 2$	A1	1.1b
	All four correct critical values $-2, -1, 0, 2$	A1	1.1b
	$\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$	M1	2.2a
		A1	2.5
		(6)	
			(6 marks)

Notes

M1	Gathers terms on one side and puts over common denominator, or multiply by $x^2(x+2)^2$ and then gather terms on one side.
M1	Factorise numerator or find roots of numerator or factorise resulting inequation into 4 factors.
A1	At least 2 correct critical values found.
A1	Exactly 4 correct critical values.
M1	Deduces that the 2 "outsides" and the "middle interval" are required. May be by sketch, number line or any other means.
A1	Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set. E.g. accept $\mathbb{R} - ([-2, -1] \cup [0, 2])$ or $\{x \in \mathbb{R} : x < -2$ or $-1 < x < 0$ or $x > 2\}$.

Q8.

Question	Scheme	Marks	AOs
	$\dfrac{x}{x^2-2x-3} \le \dfrac{1}{x+3}$		
	$\dfrac{x(x+3)-(x^2-2x-3)}{(x^2-2x-3)(x+3)} \le 0$ or $x(x-3)(x+1)(x+3)^2-(x-3)^2(x+1)^2(x+3) \le 0$ or $x(x^2-2x-3)(x+3)^2-(x^2-2x-3)^2(x+3) \le 0$	M1	2.1
	$\dfrac{5x+3}{(x-3)(x+3)(x+1)}\ \{\le 0\}$ or $(x-3)(x+1)(x+3)(5x+3)\ \{\le 0\}$	M1	1.1b
		A1	1.1b
	All three critical values $-3,\ 3,\ -1$	B1	1.1b
	Critical value $-\dfrac{3}{5}$	B1ft	1.1b
	$\{x\in\mathbb{R}: -3<x<-1\}\cup\left\{x\in\mathbb{R}: -\dfrac{3}{5}\le x<3\right\}$	M1	2.2a
		A1	2.5
		(7)	
			(7 marks)

Notes	
M1:	Gathers terms on one side and puts over a common denominator, or multiplies by $(x+1)^2(x-3)^2(x+3)^2$ (or by the equivalent $(x^2-2x-3)^2(x+3)^2$) and gathers terms onto one side
M1:	Expands and simplifies fully the numerator or takes out a factor of $(x-3)(x+1)(x+3)$ (or the equivalent $(x^2-2x-3)(x+3)$) and then simplifies fully their remaining factor
A1:	$\dfrac{5x+3}{(x-3)(x+3)(x+1)}$ or $(x-3)(x+1)(x+3)(5x+3)$
B1:	Correct critical values of -3, 3 and -1 which can be implied, e.g. from their inequalities
B1ft:	Correct critical value of $-\dfrac{3}{5}$ which can be implied, e.g. from their inequalities
Note:	**B1ft:** You can follow through their fourth factor which is in the form $(ax+b)$, $a,b\ne 0$ to give C.V. $= -\dfrac{b}{a}$, if their fourth factor is not any of either $(x-3)$, $(x+3)$ or $(x+1)$
M1:	Deduces that 2 "inside" inequalities are required with critical values in ascending order
A1:	Exactly 2 correct intervals, condoning omission of the union symbol
Note:	Also accept, e.g. • $-3<x<-1,\ -\dfrac{3}{5}\le x<3$ • $(-3,-1),\ \left[-\dfrac{3}{5},3\right)$ • $-1>x>-3,\ 3>x\ge-\dfrac{3}{5}$

Notes Continued	
Note:	Give 1st A0 for $(x^2-2x-3)(x+3)(5x+3)\ \{\le 0\}$ with no other working seen
Note:	Give 1st A1 (implied) for $(x^2-2x-3)(x+3)(5x+3)\ \{\le 0\}$ with $x=3,\ x=-1$ stated
Note:	Give 1st A0 for $\dfrac{5x+3}{(x^2-2x-3)(x+3)}\left\{\le 0\right\}$ with no other working seen
Note:	Give 1st A1 (implied) for $\dfrac{5x+3}{(x^2-2x-3)(x+3)}\left\{\le 0\right\}$ with $x=3,\ x=-1$ stated
Note:	Give 1st A0 for $\dfrac{5x+3}{x^3+x^2-9x-9}\left\{\le 0\right\}$ with no other working seen
Note:	Give 1st A1 (implied) for $\dfrac{5x+3}{x^3+x^2-9x-9}\left\{\le 0\right\}$ with $x=3,\ x=-1,\ x=-3$ stated
Note:	Allow special case final M1 for any of • $-3<x<-1$ (condoning closed inequalities or a mixture of open and closed inequalities) • $-\frac{3}{5}\le x<3$ (condoning closed inequalities or a mixture of open and closed inequalities) but ***do not allow*** M1 for any of • e.g. $-3<x<-1,\ -1<x\le-\frac{3}{5}$ ("continuing inequalities") • e.g. $-3<x<1,\ -\frac{3}{5}\le x<3$ ("overlapping inequalities")
	<u>Alternative Method</u> $x(x-3)(x+1)(x+3)^2 \le (x-3)^2(x+1)^2(x+3)$ $x^5+4x^4-6x^3-36x^2-27x \le x^5-x^4-14x^3+6x^2+45x+27$ $5x^4+8x^3-42x^2-72x-27\le 0$
Note:	$5x^4+8x^3-42x^2-72x-27\le 0$ without any other working is M1M0A0
Note:	$5x^4+8x^3-42x^2-72x-27\le 0 \Rightarrow x=-3,\ -1,\ 3$ is M1M1A1B1
Note:	$5x^4+8x^3-42x^2-72x-27\le 0 \Rightarrow x=-3,\ -1,\ 3,\ -\frac{3}{5}$ is M1M1A1B1B1